

Canonical Bases of q -Deformed Fock Spaces

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Abstract

We define a canonical basis of the q -deformed Fock space representation of the affine Lie algebra $\widehat{\mathfrak{gl}}_n$. We conjecture that the entries of the transition matrix between this basis and the natural basis of the Fock space are q -analogues of decomposition numbers of the v -Schur algebras for v specialized to a n th root of unity.

1 Introduction

The Fock space representation \mathcal{F} of $\widehat{\mathfrak{sl}}_n$ is not irreducible. Its decomposition into simple $\widehat{\mathfrak{sl}}_n$ -modules is given by [3]

$$\mathcal{F} \cong \bigoplus_{k \geq 0} M(\Lambda_0 - k\delta)^{\oplus p(k)}, \quad (1)$$

where $p(k)$ denotes the number of partitions λ of k . Hence it is not obvious to apply Kashiwara's or Lusztig's method to define a canonical basis of \mathcal{F} .

In this note, we shall rather regard \mathcal{F} as a representation of the enlarged algebra $\widehat{\mathfrak{gl}}_n$. Indeed, \mathcal{F} is a simple $\widehat{\mathfrak{gl}}_n$ -module, and a q -deformation \mathcal{F}_q of this representation has been described in [11]. One can then define a natural semi-linear involution $v \rightarrow \bar{v}$ commuting with the action of the lowering operators of \mathcal{F}_q and leaving invariant its highest weight vector. Using this involution, one obtains in an elementary way a canonical basis $\{G(\lambda)\}$ of \mathcal{F}_q . This basis can be computed explicitly. It would be interesting to compare it with the canonical basis obtained via a geometric approach by Ginzburg, Reshetikhin and Vasserot [7].

By restriction to $U_q(\widehat{\mathfrak{sl}}_n)$, the space \mathcal{F}_q decomposes similarly as

$$\mathcal{F}_q \cong \bigoplus_{k \geq 0} M_q(\Lambda_0 - k\delta)^{\oplus p(k)}, \quad (2)$$

where $M_q(\Lambda)$ denotes the simple $U_q(\widehat{\mathfrak{sl}}_n)$ -module with highest weight Λ . The $G(\mu)$ indexed by n -regular partitions μ coincide with the elements of Kashiwara's global crystal basis of the basic representation $M(\Lambda_0)$. But we insist that the rest of our basis is not compatible with the decomposition (2).

We conjecture that for $q = 1$, the coefficients of the transition matrix of our canonical basis on the natural basis of the Fock space are equal to the decomposition numbers of v -Schur algebras over a field of characteristic 0 at a n -th root of unity. A previous conjecture [12, 13] on decomposition matrices of Hecke algebras having been recently established by Ariki and by Grojnowski, we already know that the columns of the transition matrix indexed by n -regular

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partitions contain q -analogues of decomposition numbers. On the other hand, we can prove by means of a q -analogue of Steinberg's tensor product theorem that an infinite number of entries of the inverse transition matrix are q -analogues of inverse decomposition numbers.

Details and proofs will appear in a forthcoming paper.

We follow the notation of [16] for symmetric functions, and that of [18, 11] for q -wedges and Fock space representations, except for the replacement of q by q^{-1} .

2 A q -analogue of the Fock space representation of $\widehat{\mathfrak{gl}}_n$

The Lie algebra $\widehat{\mathfrak{gl}}_n$ can be regarded as the sum $\widehat{\mathfrak{sl}}_n + \mathcal{H}_n$ where \mathcal{H}_n is a Heisenberg algebra commuting with $\widehat{\mathfrak{sl}}_n$ and such that $\widehat{\mathfrak{sl}}_n \cap \mathcal{H}_n = \mathbb{C}c$, where c is the central generator [4].

The bosonic Fock space $\mathcal{F} = \mathbb{C}[x_1, x_2, \dots]$ can be interpreted as the algebra of symmetric functions in some infinite set of variables t_i by means of the correspondence $x_k = \frac{1}{k}p_k$, where $p_k = \sum_i t_i^k$ are the power sums. With this interpretation, the action of $\widehat{\mathfrak{gl}}_n$ on \mathcal{F} can be described as follows. The generator B_k of \mathcal{H}_n acts by $nk \frac{\partial}{\partial p_{nk}}$ for $k > 0$ and as the multiplication by p_{-nk} for $k < 0$. The action of the generators of $\widehat{\mathfrak{sl}}_n$ is particularly simple in the basis of Schur functions s_λ . For a node γ of the Young diagram of a partition λ , located at the intersection of the i th row and the j th column of λ , define its residue $r(\gamma) \in \{0, 1, \dots, n-1\}$ as $r(\gamma) = j - i \bmod n$. Then,

$$e_i s_\lambda = \sum_\nu s_\nu, \quad f_i s_\lambda = \sum_\mu s_\mu,$$

where ν (resp. μ) runs through all partitions obtained from λ by removing (resp. adding) a node of residue i .

The q -analogue \mathcal{F}_q of the Fock space representation of $\widehat{\mathfrak{sl}}_n$, in the form of a Fock space representation of the quantized enveloping algebra $U_q(\widehat{\mathfrak{sl}}_n)$, has been constructed by Hayashi [8] and further investigated by Misra and Miwa [17] who constructed the crystal basis. Recently, Kashiwara, Miwa and Stern [11] have shown that the action of $U_q(\widehat{\mathfrak{sl}}'_n)$ on \mathcal{F}_q is centralized by a Heisenberg algebra \mathcal{H}_n^q . Let \mathbf{U} be the subalgebra of $\text{End}(\mathcal{F}_q)$ generated by these actions of $U_q(\widehat{\mathfrak{sl}}_n)$ and \mathcal{H}_n^q (it would be interesting to compare \mathbf{U} with the standard q -deformation $U_q(\widehat{\mathfrak{gl}}_n)$ considered in [5, 7]). The Fock space is an irreducible \mathbf{U} -module, and the canonical basis constructed in this paper will be adapted to this representation.

The Fock space representation of $U_q(\widehat{\mathfrak{sl}}_n)$ can be described as follows (note that our conventions are slightly different from those of [17] and [11]). Let

$$\mathcal{F}_q = \bigoplus_\lambda \mathbb{Q}(q)|\lambda\rangle$$

be the $\mathbb{Q}(q)$ vector space with basis $|\lambda\rangle$ indexed by the set of all partitions. Let λ and μ be two partitions such that μ is obtained from λ by adding a node γ of residue i . Let $I_i(\lambda)$ be the number of indent i -nodes of λ , $R_i(\lambda)$ the number of its removable i -nodes, $I_i^l(\lambda, \mu)$ (resp. $R_i^l(\lambda, \mu)$) the number of indent i -nodes (resp. of removable i -nodes) situated to the left of γ (γ not included), and similarly, let $I_i^r(\lambda, \mu)$ and $R_i^r(\lambda, \mu)$ be the corresponding numbers for nodes located on the right of γ . Set $N_i(\lambda) = I_i(\lambda) - R_i(\lambda)$, $N_i^l(\lambda, \mu) = I_i^l(\lambda, \mu) - R_i^l(\lambda, \mu)$ and $N_i^r(\lambda, \mu) = I_i^r(\lambda, \mu) - R_i^r(\lambda, \mu)$. Then,

$$f_i |\lambda\rangle = \sum_\mu q^{N_i^r(\lambda, \mu)} |\mu\rangle, \quad e_i |\mu\rangle = \sum_\lambda q^{N_i^l(\lambda, \mu)} |\lambda\rangle \quad (3)$$

where in each case the sum is over all partitions such that μ/λ is a i -node,

$$q^{h_i} |\lambda\rangle = q^{N_i(\lambda)} |\lambda\rangle \quad \text{and} \quad q^D |\lambda\rangle = q^{-N^0(\lambda)} |\lambda\rangle \quad (4)$$

where D is the degree generator and $N^0(\lambda)$ the total number of 0-nodes of λ .

The Fock representation of the Heisenberg algebra \mathcal{H}_n^q of [11] is defined by means of Stern's construction of semi-infinite q -wedges [18]. The basis vector $|\lambda\rangle$ is interpreted as the semi-infinite wedge $u_I = u_{i_1} \wedge u_{i_2} \wedge \cdots$ where the sequence I is defined by $i_k = \lambda_k - k + 1$. With our conventions, the commutation relations for q -wedges are the following. Suppose that $\ell < m$ and that $\ell - m \bmod n = i$. If $i = 0$ then $u_\ell \wedge u_m = -u_m \wedge u_\ell$ and otherwise,

$$u_\ell \wedge u_m = -q^{-1} u_m \wedge u_\ell + (q^{-2} - 1) (u_{m-i} \wedge u_{\ell+i} - q^{-1} u_{m-n} \wedge u_{\ell+n} + q^{-2} u_{m-n-i} \wedge u_{m+n+i} - \cdots)$$

where in the last sum one retains only the normally ordered terms.

Then, the generator B_k of \mathcal{H}_n^q acts on u_I by

$$B_k u_I = \sum_{i \geq 1} u_{I+kn\delta^i} \quad (5)$$

where δ^i is the sequence $(\delta_j^i)_{j \geq 1}$ (Kronecker symbols). The semi-infinite wedge u_I will be called the fermionic realization of the basis vector $|\lambda\rangle$. These operators verify the relations [11]

$$[B_k, B_\ell] = k \frac{1 - q^{-2nk}}{1 - q^{-2k}} \delta_{k+\ell, 0} . \quad (6)$$

The action of \mathcal{H}_n^q in the bosonic picture is better described in terms of other operators $U_k, V_k \in U(\mathcal{H}_n^q)$ [14]. Recall that for $k > 0$, B_{-k} is a q -analogue of multiplication by $p_{kn} = p_k(t_1^n, t_2^n, \dots) = \psi^n(p_k)$, where ψ_n is the ring homomorphism raising the variables to the n th power. On the graphical representation by Young diagrams, multiplication by the complete homogeneous functions $\psi_n(h_k) = h_k(t_1^n, t_2^n, \dots)$ of n th powers has a simple combinatorial description. Let

$$V_k = \sum_{m_1+2m_2+\dots+km_k=k} \frac{1}{m_1!m_2!\dots m_k!} \left(\frac{B_{-1}}{1}\right)^{m_1} \left(\frac{B_{-2}}{2}\right)^{m_2} \cdots \left(\frac{B_{-k}}{k}\right)^{m_k} \quad (7)$$

be the q -analogue of the multiplication operator by $\psi^n(h_k)$. Then,

$$V_k |\lambda\rangle = \sum q^{-\mathbf{h}(\mu/\lambda)} |\mu\rangle \quad (8)$$

where the sum is over all partitions μ such that μ/λ is a horizontal n -ribbon strip of weight k , and

$$\mathbf{h}(\mu/\lambda) = \sum_R (\text{ht}(R) - 1)$$

where the sum is over all the n -ribbons R tiling μ/λ , $\text{ht}(R)$ being the height of the ribbon R (see [14]).

The scalar product on \mathcal{F}_q is defined by $\langle \lambda | \mu \rangle = \delta_{\lambda\mu}$. The adjoint operator U_k of V_k acts by

$$U_k |\mu\rangle = \sum q^{-\mathbf{h}(\mu/\lambda)} |\lambda\rangle \quad (9)$$

where the sum is over all partitions λ such that μ/λ is a horizontal n -ribbon strip of weight k .

Identifying \mathcal{F}_q with $\mathbb{Q}(q) \otimes \text{Sym}$ by setting $|\lambda\rangle = s_\lambda$, one can define a linear operator

$$\psi_q^n : \mathcal{F}_q \longrightarrow \mathcal{F}_q$$

by specifying the image of the basis (h_λ) as

$$\psi_q^n(h_\lambda) = V_{\lambda_1} V_{\lambda_2} \cdots V_{\lambda_r} |\emptyset\rangle . \quad (10)$$

Then, the image $\{\psi_q^n(g_\lambda)\}$ of any basis $\{g_\lambda\}$ of symmetric functions will be a basis of the space of $U_q(\widehat{\mathfrak{sl}}_n)$ -highest weight vectors in \mathcal{F}_q .

3 An involution of the Fock space

Let \mathcal{J} be the set of decreasing sequences $I = (i_1, i_2, \dots)$ such that $i_k = -k+1$ for k large enough. Then $\{u_I \mid I \in \mathcal{J}\}$ is the standard basis of \mathcal{F}_q . For $m \geq 0$, denote by \mathcal{J}_m the subset of \mathcal{J} consisting of those I such that $\sum_k (i_k + k - 1) = m$. Let $I \in \mathcal{J}_m$, and let u_I be the associated basis vector of \mathcal{F}_q . We denote by $\alpha_{n,k}(I)$ the number of pairs (r, s) with $1 \leq r < s \leq k$ and $r - s \not\equiv 0 \pmod n$.

Proposition 3.1 *For $k \geq m$, the q -wedge*

$$\overline{u_I} = (-1)^{\binom{k}{2}} q^{\alpha_{n,k}(I)} u_{i_k} \wedge u_{i_{k-1}} \wedge \dots \wedge u_{i_1} \wedge u_{i_{k+1}} \wedge u_{i_{k+2}} \wedge \dots$$

is independent of k .

Define a semi-linear map $v \mapsto \overline{v}$ in \mathcal{F}_q by

$$\sum_{I \in \mathcal{J}} \overline{\varphi_I(q) u_I} = \sum_{I \in \mathcal{J}} \varphi_I(q^{-1}) \overline{u_I}. \quad (11)$$

Theorem 3.2 (i) $v \mapsto \overline{v}$ is an involution of \mathcal{F}_q .

(ii) $\overline{f_i v} = f_i \overline{v}$ and $\overline{B_{-k} v} = B_{-k} \overline{v}$, ($v \in \mathcal{F}_q$, $i \in \{0, \dots, n-1\}$, $k > 0$).

We note that there is a unique semi-linear map satisfying (ii) and $\overline{|\emptyset\rangle} = |\emptyset\rangle$. This implies that the restriction of the involution $v \mapsto \overline{v}$ to the subspace $M(\Lambda_0)$ of \mathcal{F}_q coincides with the usual involution in terms of which the global crystal basis of $M(\Lambda_0)$ is defined.

Let $\mu \vdash m$. Set

$$\overline{|\mu\rangle} = \sum_{\lambda \vdash m} a_{\lambda\mu}(q) |\lambda\rangle.$$

Theorem 3.3 (i) $a_{\lambda\mu}(q) \in \mathbb{Z}[q, q^{-1}]$.

(ii) $a_{\lambda\mu}(q) = 0$ unless $\lambda \leq \mu$ and λ, μ have the same n -core.

(iii) $a_{\lambda\lambda}(q) = 1$.

(iv) $a_{\lambda\mu}(q) = a_{\mu'\lambda'}(q)$.

For $n = 2$, the matrices $\mathbf{A}_m(q) = [a_{\lambda\mu}(q)]_{\lambda, \mu \vdash m}$ for $m = 2, 3, 4$ are

$$\begin{bmatrix} 1 & 0 \\ q - q^{-1} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ q - q^{-1} & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ q - q^{-1} & 1 & 0 & 0 & 0 \\ -1 + q^{-2} & q - q^{-1} & 1 & 0 & 0 \\ 0 & q^2 - 1 & q - q^{-1} & 1 & 0 \\ q^2 - 1 & 0 & -1 + q^{-2} & q - q^{-1} & 1 \end{bmatrix}$$

(partitions are ordered in reverse lexicographic order, e.g., for $m = 4$, $(4), (31), (22), (211), (1111)$).

4 Canonical bases

Let L (resp. L^-) be the $\mathbb{Z}[q]$ (resp. $\mathbb{Z}[q^{-1}]$)-lattice in \mathcal{F}_q with basis $\{|\lambda\rangle\}$. Using Theorems 3.2 and 3.3 one can construct “IC-bases of \mathcal{F}_q ”, in the terminology of Du [6] (see also [15], 7.10).

Theorem 4.1 *There exist bases $\{G(\lambda)\}$, $\{G^-(\lambda)\}$ of \mathcal{F}_q characterized by:*

- (i) $\overline{G(\lambda)} = G(\lambda), \quad \overline{G^-(\lambda)} = G^-(\lambda),$
- (ii) $G(\lambda) \equiv |\lambda\rangle \pmod{qL}, \quad G^-(\lambda) \equiv |\lambda\rangle \pmod{q^{-1}L^-}.$

Set

$$G(\mu) = \sum_{\lambda} d_{\lambda\mu}(q) |\lambda\rangle, \quad G^-(\lambda) = \sum_{\mu} e_{\lambda\mu}(q) |\mu\rangle.$$

Then $d_{\lambda\mu}(q) \in \mathbb{Z}[q]$, $e_{\lambda\mu}(q) \in \mathbb{Z}[q^{-1}]$, and these polynomials are nonzero only if λ, μ have the same n -core. Moreover, $d_{\lambda\lambda}(q) = e_{\lambda\lambda}(q) = 1$, and $d_{\lambda\mu}(q) = 0$ unless $\lambda \leq \mu$, $e_{\lambda\mu}(q) = 0$ unless $\mu \leq \lambda$. Also, $\{G(\lambda) \mid \lambda \text{ } n\text{-regular}\}$ coincides with the lower crystal basis of the basic representation $M(\Lambda_0)$ of $U_q(\widehat{\mathfrak{sl}}_n)$.

Let $\{G^\dagger(\lambda)\}$ be the adjoint basis of $\{G(\lambda)\}$. It follows from Theorem 3.3 (iv) that

$$G^\dagger(\lambda)' = G^-(\lambda')$$

where $v \mapsto v'$ denotes the semi-linear involution of \mathcal{F}_q defined by $|\lambda\rangle' = |\lambda'\rangle$.

We set

$$G^\dagger(\lambda) = \sum_{\mu} c_{\lambda\mu}(q) |\mu\rangle,$$

so that $c_{\lambda\mu}(q) = e_{\lambda'\mu'}(q^{-1})$ and $[c_{\lambda\mu}(q)] = [d_{\lambda\mu}(q)]^{-1}$.

For $n = 2$ and $m \leq 6$, the matrices $\mathbf{D}_m(q) = [d_{\lambda\mu}(q)]_{\lambda, \mu \vdash m}$ are

$$\begin{array}{ccc} 2 & 1 & 0 \\ 11 & q & 1 \end{array} \quad \begin{array}{cccc} 3 & 1 & 0 & 0 \\ 21 & 0 & 1 & 0 \\ 111 & q & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} 4 & 1 & 0 & 0 & 0 & 0 \\ 31 & q & 1 & 0 & 0 & 0 \\ 22 & 0 & q & 1 & 0 & 0 \\ 211 & q & q^2 & q & 1 & 0 \\ 1111 & q^2 & 0 & 0 & q & 1 \end{array} \quad \begin{array}{cccccccc} 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 41 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 32 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 311 & q & 0 & q & 1 & 0 & 0 & 0 \\ 221 & 0 & 0 & q^2 & q & 1 & 0 & 0 \\ 2111 & 0 & q & 0 & 0 & 0 & 1 & 0 \\ 11111 & q^2 & 0 & 0 & q & 0 & 0 & 1 \end{array}$$

6	1	0	0	0	0	0	0	0	0	0	0
51	q	1	0	0	0	0	0	0	0	0	0
42	0	q	1	0	0	0	0	0	0	0	0
411	q	q^2	q	1	0	0	0	0	0	0	0
33	0	0	q	0	1	0	0	0	0	0	0
321	0	0	0	0	0	1	0	0	0	0	0
3111	q^2	q	q^2	q	q	0	1	0	0	0	0
222	0	0	q^2	q	q	0	0	1	0	0	0
2211	0	q^2	q^3	q^2	q^2	0	q	q	1	0	0
21111	q^2	q^3	0	q	0	0	q^2	0	q	1	0
111111	q^3	0	0	q^2	0	0	0	0	0	q	1

5 A q -analogue of Steinberg's tensor product theorem

Let α be a partition of r . Define an element S_α of \mathbf{U} by

$$S_\alpha = \sum_{\beta \vdash r} \frac{\chi_\beta^\alpha}{z_\beta} B_{-\beta_1} B_{-\beta_2} \cdots B_{-\beta_k} \quad (12)$$

where $\beta = (\beta_1, \dots, \beta_k) = (1^{m_1} 2^{m_2} \dots r^{m_r})$, $z_\beta = 1^{m_1} m_1! \cdots r^{m_r} m_r!$ and χ_β^α is the value of the irreducible character χ^α of the symmetric group on a permutation of cycle type β .

Writing $V_\mu = V_{\mu_1} V_{\mu_2} \cdots V_{\mu_r}$, one has

$$S_\alpha = \sum_{\mu \vdash r} \kappa_{\alpha\mu} V_\mu,$$

where the $\kappa_{\alpha\mu}$ denote the entries of the inverse Kostka matrix. Hence, using (8), one can describe the action of S_α on Young diagrams.

The operator S_α is a q -analogue of the multiplication by the plethysm $\psi^n(s_\alpha)$ in the ring of symmetric functions.

Let λ be a partition such that λ' is n -singular. We can write $\lambda = \mu + n\alpha$ where $n\alpha = (n\alpha_1, n\alpha_2, \dots)$ and μ' is n -regular.

Theorem 5.1 $G^-(\lambda) = S_\alpha(G^-(\mu)).$

This reduces the computation of $\{G^-(\lambda)\}$ to that of the subfamily $\{G^-(\mu) \mid \mu \text{ } n\text{-regular}\}.$

Let \mathbf{D}_m denote the decomposition matrix of the v -Schur algebra \mathcal{S}_m over a field of characteristic 0, for v a primitive n -th root of unity. We use the notational convention of James [9], that is, the rows and columns of \mathbf{D}_m are indexed in such a way that \mathbf{D}_m is the matrix Δ_m of [9] for big p .

Conjecture 5.2 *The matrix $\mathbf{D}_m(1)$ is equal to the decomposition matrix \mathbf{D}_m .*

This conjecture, which generalizes Conjecture 6.9 of [13], is already verified to a large extent. Indeed, on the one hand Ariki [1] and Grojnowski have verified independently our previous conjecture, which means that the $d_{\lambda\mu}(1)$ for μ n -regular are equal to the corresponding decomposition numbers of \mathcal{S}_m . On the other hand, it follows from results of James [9] that the entries of the inverse matrices \mathbf{D}_m^{-1} satisfy the same properties as those deduced from Theorem 5.1 for the coefficients $c_{\lambda\mu}(1)$. Since we have verified, using the tables of [9], that $\mathbf{D}_m^{-1} = [c_{\lambda\mu}(1)]$ for

$m \leq 10$ (any n), we deduce that an infinite number of $c_{\lambda\mu}(1)$ coincide with the corresponding entries of \mathbf{D}_m^{-1} .

The following refined conjecture has also been checked for small m . It generalizes the conjecture of Section 9 of [13], due to Rouquier. Let $(W(\lambda)^i)$ be the Jantzen filtration of the Weyl module $W(\lambda)$ for the v -Schur algebra \mathcal{S}_m , and let $L(\mu)$ be the irreducible module corresponding to μ [10].

Conjecture 5.3 *Let λ, μ be partitions of m . Then,*

$$d_{\lambda'\mu'}(q) = \sum_{i \geq 0} [W(\lambda)^i / W(\lambda)^{i+1} : L(\mu)] q^i.$$

Finally, we note the following combinatorial description of some polynomials $e_{\lambda\mu}(q)$ in the case $n = 2$, which proves that they are equal (up to sign and the replacement of q by q^{-2}) to the q -analogues of Littlewood-Richardson coefficients introduced in [2].

Theorem 5.4 *Let $n = 2$. One has*

$$e_{2\lambda,\mu}(q) = \varepsilon_2(\mu) \sum_{T \in \text{Yam}_2(\mu,\lambda)} q^{-2\text{spin}(T)}$$

where $\text{Yam}_2(\mu, \lambda)$ denotes the set of Yamanouchi domino tableaux of shape μ and weight λ , and $\varepsilon_2(\mu)$ is the 2-sign of μ .

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